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13. ABSTRACT (Max) <p>Cellular Nonlinear Networks (CNNs) are large arrays of nonlinear circuits coupled to their immediate neighbors. During this funding period we have made many advances in understanding the pattern-forming dynamics of such circuits and their relationship to problems in physics and biology, we have explicated the image processing capabilities of such CNNs, including spatial filtering and multiscale analysis, and finally we have obtained rigorous mathematical results concerning the dynamic behavior of simple CNN arrays.</p> <p>Large arrays of <i>complex</i> cells have elsewhere been shown to demonstrate interesting pattern forming behaviors—such as the reaction-diffusion systems of Turing, the propagation of autowaves, and the Ising spin system. We have shown that the <i>simple first-order</i> CNN is capable of exhibiting the essential features found in these systems. And, due to the continuous-time nonlinear dynamics and general neighborhood weights the patterns formed by the CNN proved to be a study in their own right.</p> <p>The linear spatial convolution operation is essential in all manner of nonlinear and linear image processing algorithms. Under this funding, we demonstrated an approach by which any arbitrary FIR filter could be implemented in a robust and straightforward manner via a CNN Universal Machine (CNUM) algorithm – by using only a standard 3×3 B-template. In addition, a general canonical form was proposed for such CNN linear convolutions, which also takes into account the use of an A-template and previous approaches to convolution.</p> <p>Finally, we initiated an investigation into a rigorous analysis of the CNN dynamics as a function of the template parameters. The first theorems have been obtained for the two-cell CNN case. We expect generalizations of such results to verify the proper behavior of many CNN image processing templates.</p>				
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Pattern Formation Properties of Cellular Neural Networks

Pattern formation in the CNN is a result of the mechanism of symmetry breaking around an unstable equilibrium. The symmetry can be broken by either small random noise, a 'seed' or site defect, or a systematic disturbance. The parameters which affect the type of pattern formed are the bias and interconnection weights.

Random Perturbation

When the symmetry breaking is introduced as a small random perturbation, our research has led us to identify the following three epochs of behavior in the time formation of patterns.

- Linear System leading to noise shaping.
- Separation of Spatial Frequencies and Phases leading to meta-stability.
- Boundary Negotiation leading to stability.

Various observations and theoretical results have been made for each of these epochs, which are summarized below:

Linear System

If the system is halted at the moment when the first cell of the array enters the saturation region, the linear system theory will be exact. The states at this time will be the result of a linear filtering operation which can be found in terms of the template weights. Such a filtering operation can be understood to enhance certain spatial frequencies while suppressing others.

Motif Separation

In regions where a significant number of cells have reached saturation, the linear analysis does not even hold approximately. At this point, arbitrary combinations of the unstable modes can no longer be maintained. The states begin to separate themselves into regions with motifs that are at least locally stable. These regions grow until their boundaries meet with other regions of different motif. At this point the system is considered to be meta-stable in the sense that most cells are unchanging in time (the interior cells of the regions) and only the boundaries between regions are moving.

It is difficult to determine what the possible meta-stable motifs are. We have shown that if any combination of equally unstable modes has a purely binary representation, then in the saturation region, that motif is stable, at least in the sense that a big enough patch of it will not change unless outside influences force it to. It is not certain that a finite array containing this motif would be stable, however.

For a 3×3 template of weights (i.e. immediate neighbors) we have fully characterized the stable motifs. In addition, simple inequalities allow the determination of the dominant stable motif, which will generally arise from random initial conditions.

Boundary Negotiation

Even though the array has separated into regions that are locally stable, the boundary between them may prove to be unstable. Thus begins a long process where most of the cells in the array are not changing in time, but slowly the boundaries are negotiated. In some cases there may not be

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any acceptable boundary, and eventually one motif must cover the whole array. The whole process may take many order of magnitudes longer than the first two steps.

Bias

We have studied some of the effects of using the bias term in the dynamics. The bias changes the allowed meta-stable motifs. For instance, we have used this fact to demonstrate the formation of 'stripes' of a tiger when no bias is used and the 'spots' of a leopard when bias is added. Varying the bias in time can simulate some of the effects of striping patterns in the angelfish. Finally, we have begun investigations into the use of space-varying bias for image processing applications such as fingerprint enhancement.

Site Defects

Finally, we have investigated the mechanism involved in the production of spiral and target patterns in the CNN by using 'seeds' to break the unstable equilibrium. When one site of the array is disturbed from equilibrium, a pattern begins propagating outward. This will usually take the form of a circular blob containing the dominant motif. However, when more than one seed is used in the same array, these motifs must join up in a consistent way, which can cause the formation of spirals. The patterns formed are reminiscent of the aggregate growths of bacteria colonies, for instance. Furthermore, we have discovered template values which exhibit dendritic growth from a site defect, a previously unknown phenomena in the first-order CNN cell array.

Linear Spatial Convolution

As linear filtering is the workhorse of image processing algorithms, the question of whether the CNUM can implement any convolution and how to do so is important. We have shown how these methods can be applied to multiscale image processing.

It has long been understood that the B-template of the simple CNN performs a correlation (reflected convolution) with the input image. However, due to implementation concerns the B-template, and therefore the convolution kernel, is restricted to be small in size – typically 3×3 . By also using the A-template, or by cascading B-template operations, large kernel convolutions can be performed but the coefficients of the impulse response cannot be arbitrarily specified. However, the need to exactly specify a large convolution mask arises in many situations, such as template matching for object detection, dilation or erosion by large structuring elements, and interpolation.

Canonical Form for General FIR Filtering

We have developed a canonical form for the possible spatial linear filtering operations which can be performed on the CNN Universal Machine, when running a finite algorithm using single-transient CNN spatial convolution and image addition. Convolution by arbitrary kernels of any finite size can be shown to be implementable by algorithms of this form using only 3×3 templates. Equivalently it can be considered a general form capable of implementing arbitrarily large B-template effects by using only 3×3 connectivity.

Previous methods for performing arbitrary convolution, which involved either summations of multiple applications of small B-templates or partitioning and shifting can be unified under this form. When implementing a desired convolution with this form the choice of templates is under-determined and either of these previous methods can be used to demonstrate completeness. We

have produced a CNUM algorithm for implementing the partition-shift approach with special concern given to practical issues of addition and accuracy. It was presented both as a constructive proof of the general convolution capabilities of the CNUM and as a practical approach in some cases. The unified form includes a broader class of implementations than, for instance, those that use the A-template for IIR filtering, by combining multiple single-transient filtering operations through image addition and cascaded convolution (i.e. serial-parallel forms).

Multiscale Analysis

As an interesting application of linear filtering on the CNN Universal Machine, our work has also included the investigation of some methods for generating multi-scale representations of gray-scale imagery on the CNUM. The method uses time-indexed CNN spatial filtering operations (analogous to 'stopped diffusion'), which are then combined under the canonical form by image subtractions and cascaded convolutions to produce the desired filtering operation.

Size and relative size of objects are among the basic variables in image analysis, it is essential therefore for any image processing system to be able to extract scale related information. A powerful way to obtain scale-space information is through generating and analyzing multi-scale representations of the input data. Features at or below a given scale are normally extracted by convolution operators where the relative size of the image and the convolution kernel determines whether the feature will be extracted or suppressed.

The CNN Universal Machine, can be used for multi-scale processing by making use of the dynamics of CNN analog computation and combining the results. By using the CNN to approximate the heat equation, the scale parameter of the linear diffusion scale-space is translated into the running time of a transient on the CNN. Scale-space filters of theoretically arbitrary size can be implemented by this method. These filtered images can then be combined in a CNUM context to produce images containing only selected feature scales. Such a solution allows for arbitrary quantization of the scale parameter as opposed to the coarse 1:2 ratio resulting from subsampling done by discarding every other sample.

The investigated approach provides for easy generation of various multi-scale representations. In particular, it has been shown how the CNUM can be used to approximate Gaussian and Laplacian multi-scale decompositions, as well as approximating the Continuous Wavelet Transform.

Mathematical Study of CNN Dynamics

During this period, we have also initiated a research effort to provide a rigorous mathematical basis for the CNN paradigm, especially for image processing applications. The questions which we would like to eventually answer include:

1. When should we expect a CNN to have stable dynamics, i.e. when will the system settle to a steady state? This depends on the templates, the number of nodes in the system, and the value of the bias. Previously, we have obtained some results in this area (e.g. symmetric templates, cooperative templates), but some of the basic cases are still not fully understood.
2. How does the qualitative behavior of the system change as the parameters are varied? For instance, what happens to an equilibrium as it is smoothly pushed out of the central linear region of state space? We believe that ideas from the field of bifurcation theory will be useful in answering these types of questions.

3. Is there a systematic way to determine the functionality of a CNN from the system equations or template values? This is a very fundamental and important question, not only to prove that a particular template has the desired behavior, but because even a partial answer can be very useful in designing templates for various applications. We envision the development of a 'template calculus' which allows us to design CNNs with complex functionalities from combinations of simple templates.

In our attempt to address such questions, we have obtained the following initial results. The two node cellular neural network (CNN) is the system described by the equations:

$$\begin{aligned}\dot{x} &= -x + b\sigma(x) + c\sigma(y) + i_x \\ \dot{y} &= -y + b\sigma(y) + a\sigma(x) + i_y\end{aligned}$$

where x and y are the activations of the cells, a , b , and c are parameters (often referred to as the "A template"), i_x and i_y are the node inputs, and $\sigma(s)$ is the function

$$\sigma(s) = \begin{cases} -1 & \text{if } s \leq -1 \\ s & \text{if } -1 \leq s \leq 1 \\ 1 & \text{if } 1 \leq s. \end{cases}$$

Thus a particular CNN is one where a , b , and c are chosen and fixed, and x and y are allowed to vary with time, subject to the differential equations above. In general, the inputs, i_x and i_y , may be either constant or allowed to vary with time. For the theorems stated below, $i_x = 0$ and $i_y = 0$.

There is much experimental data about the behavior of CNN's, for two nodes or many nodes, but the mathematical model of them is not well understood for many possible sets of parameters, either with or without external inputs (i_x and i_y). There are some previous results which characterize the behavior of CNN's for various parameter values (for instance when $a = c$). We have extended this set of results as follows:

Theorem 1 *For a two node CNN with no inputs, if $b < 1$ then the system is convergent, with a global attractor at $(0, 0)$.*

Theorem 2 *For a two node CNN with no inputs, in the parameter range given by $b > 1$, $ac < 0$, $b < \max\{c + 1, -c + 1\}$, and $b < \max\{a + 1, -a + 1\}$, there are at most two cycles which lie entirely outside the box $|x| < 1, |y| < 1$.*

The first result is an application of the divergence theorem. Unfortunately, this application is only valid in two dimensions. However, the knowledge that these systems are convergent may help to conclude that higher dimensional CNN's are also convergent when $b < 1$. The similarity of the behavior for all of these parameter values has led us to look for a Lyapunov function which will be valid whenever $b < 1$. This line of inquiry looks promising. If such a Lyapunov function is found, it is likely that it will be generalizable to higher dimensional systems.

For the second result, the piecewise linear structure of the equations was exploited. Since the (x, y) plane may be divided into regions where the differential equations are linear, solutions may be written down which are valid as long as a trajectory stays within one of these regions. A complete solution may then (in theory) be constructed through any point (x_0, y_0) .

Unfortunately, there is a serious barrier to constructing the solution through a particular initial condition as a function of the parameters. To construct a solution, one starts with the knowledge of the general solutions to the differential equations that are valid in each region of the phase space

(as mentioned above, the equations are linear in each region). Then one “steps” between particular solutions of these equations to construct a complete solution. This process involves finding the coordinates of the point where a particular solution through (x_0, y_0) intersects the boundary of a region. For most choices of (x_0, y_0) , this would involve finding the zeros of a polynomial of degree $|b - 1|$, and so this procedure cannot be carried out in general.

However, for some initial conditions, the equation is a polynomial of degree two rather than $|b - 1|$, and can be found in closed form. This makes it possible to find the Poincaré (first return) map for the ray $x = 1, y > 1$, but only for points whose trajectories remain outside the center box. If one or more cycles exists entirely outside the center box (where $|x| < 1, |y| < 1$) for the parameter values described in Theorem 2, then they will be fixed points of this map. The map is a composition of maps of the form $(py + q)/(ry + t)$, so we may conclude that it has at most two fixed points, and thus at most two periodic orbits (which gives Theorem 2).

The same methods which give Theorem 1 can be used to disallow some types of cycles for other parameter values, but we do not yet fully understand the possibilities in these cases. We are currently using these and other methods to characterize behavior in the remaining cases as fully as possible. Other work in progress includes calculating the boundaries of basins of attractions for systems with more than two nodes. This is computationally prohibitive for large systems, since the calculation involves solving 3^n differential equations, but knowledge of how complicated the basin boundaries may be for smaller systems is likely to help with the understanding and design of larger cellular neural networks.

List of New Publications

- [1] K. R. Crounse. *Image Processing Techniques for Cellular Neural Network Hardware*. PhD thesis, University of California, Berkeley, June 1997.
- [2] K. R. Crounse, L. O. Chua, P. Thiran, and G. Setti. Characterization and dynamics of pattern formation in Cellular Neural Networks. *International Journal of Bifurcation and Chaos*, 6(9):1703–1724, Sept. 1996.